DILVCIDATIONES ANALYSEOS FINITORVM KÆSTNERIANÆ

QVAS
PRÆSIDE
IOANNE KIES
PHYSICES ET MATHESEOS PROF. P. O.
FACVLTATIS PHILOSOPHICÆ h. t. DECANO
PVBLICE TVEBITVR
Diebus OCTOBRIS ANNI MDCCLXII.
FRIDERICVS LVDOVICVS HELLER,
Canstadiensis.
LAUREÆ SECVNDÆ CANDIDATVS, ET SERENISSIMI STIPENDIARIVS.

Tubingæ,
TYPIS IO. AD. SIGMVNDI.

2

VIRO
PERILLVSTRI atque EXCELLENTISSIMO
DOMINO
ERNESTO HENRICO MYLIO,
JVRIS VTRIVSQVE DOCTORI, ET SERENISSIMI DOMINI DVCIS A LEGATIONIBVS SECRETIORIBVS,
rel. rel.
MÆCENATI ac PATRONO SVO
ÆTERNO SVBMISSIONIS AC REVERENTIÆ CVLTV MAXIME SVSPICIENDO,
SPECIMEN HOC ACADEMICVM
EA, QVA PAR EST, ANIMI DEVOTIONE, DICARE,
INSIMVLQVE
SE, SVAQVE OMNIA SVBMISSISSIME COMMENDARE
VOLVIT, DEBVIT
PERILLVSTRIS NOMINIS

Cultor ſubmiſſiſſimus,
RESPONDENS.

Quum hac æstate Elementa *Analyseos finitorum Kæstneriana* satis magno Auditorum numero explicabam, accidit subinde, ut evolutio distincta quorundam problematum, & ipsa calculi administratio nimiam horarum seriem insumeret, & diutius nos teneret, quam quidem ab initio nobis erat propositum, consultum duxi, in his paginis ea uberius exponere, quæ Celeberrimus *Kæstnerus* succincte, ne liber ejus in nimiam molem excresceret, & breviter exhibuit, habebunt ita Honoratissimi Domini Auditores instrumentum quoddam utile, quo in repetendis prælectionibus meis, & secandis qui in profundissime conscripto isto libello obvenire solent nodis commode uti possunt, mihi vero idem labor inposterum inserviet, si de novo istius *Analyseos* interpres fuero, ut cum discentibus quibus hæc specimina ad manus sunt, celerius progredi liceat.

Elementa Analyſeos finitorum Kæſtneriana

§. 13.

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \&c.$$

$$\frac{1}{x+1} = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} - \frac{1}{x^4} + \frac{1}{x^5} - \&c.$$

Ergo

$$1 - x + x^2 - x^3 + x^4 - x^5 + \&c. = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} - \frac{1}{x^4} + \frac{1}{x^5} - \&c.$$

§. 98.

Sit progreſſio geometrica

$a \quad ea \quad e^2a \quad e^3a \quad e^4a \quad e^5a,$

in qua terminus primus a, & exponens e, hæc ſeries ſi per exponentem e unitate mulctatum i. e. per $e - 1$ multiplicetur, productum erit $e^6a - a$, adeoque ſi $e^6a - a$ per $e - 1$ dividatur, prodibit iterum ſeries $a \quad ea \quad e^2a \quad e^3a \quad e^4a \quad e^5a = \frac{e^6a - a}{e - 1}$

§. 110.

$$\frac{1}{y} - \frac{1}{y^2} + \frac{1}{y^3} - \frac{1}{y^4} + \frac{1}{y^5} - \frac{1}{y^6} + \&c.$$

in hac ſerie exponens eſt $-\frac{1}{y}$ & ea abit in infinitum, hinc ſumma erit

$$\frac{1}{y} : 1 + \frac{1}{y} = \frac{1}{1+y}$$

§. 163.

In Figura 4. ubi DF ſecat lineam BC, notetur interſectio litera I, & habebitur, poſitis denominationibus *Kæſtnerianis*

$CE : EB = CF : FI.$

$\surd(r^2 - \frac{1}{4}k^2) : \frac{1}{2}k = \surd(r^2 - \frac{1}{4}c^2) : FI$, ergo

$$FI = \frac{\frac{1}{2}k . \surd(r^2 - \frac{1}{4}c^2)}{\surd(r^2 - \frac{1}{4}k^2)}$$

$$DI = \frac{1}{2}c - \frac{\frac{1}{2}k\surd(r^2 - \frac{1}{4}c^2)}{\surd(r^2 - \frac{1}{4}k^2)}$$

$CE : CB = CF : CI$

$\surd(r^2 - \frac{1}{4}k^2) : r = \surd(r^2 - \frac{1}{4}c^2) : CI$ hinc

$$CI = \frac{r\surd(r^2 - \frac{1}{4}c^2)}{\surd(r^2 - \frac{1}{4}k^2)}$$

$CI : DI = CF : DG$

$$\frac{r\surd(r^2 - \frac{1}{4}c^2)}{\surd(r^2 - \frac{1}{4}k^2)} : \frac{1}{2}c - \frac{\frac{1}{2}k\surd(r^2 - \frac{1}{4}c^2)}{\surd(r^2 - \frac{1}{4}k^2)} = \surd(r^2 - \frac{1}{4}c^2) : DG$$

hinc $DG = \dfrac{\frac{1}{2}c\surd(r^2 - \frac{1}{4}k^2) - \frac{1}{2}k\surd(r^2 - \frac{1}{4}c^2)}{r}$

& $DK = \dfrac{c\surd(r^2 - \frac{1}{4}k^2) - k(\surd r^2 - \frac{1}{4}c^2)}{r}$

$$DK = \frac{c\surd(4r^2 - k^2) - k\surd(4r^2 - c^2)}{2r}$$

§. 164.

$EC : BC = DG . DI$

$\frac{1}{2}\surd(4r^2 - k^2) : r = \frac{1}{2}b : \dfrac{rb}{\surd(4r^2 - k^2)}$

$EC : BE = DG : GI$

$\frac{1}{2}\surd(4r^2 - k^2) : \frac{1}{2}k = \frac{1}{2}b : \dfrac{bk}{2\surd(4r^2 - k^2)}$

$GC - GI = CI = \frac{1}{2}\surd(4r^2 - b^2) - \dfrac{bk}{2\surd(4r^2 - k^2)}$

$BC : BE = IC : IF$

$$r : \tfrac{1}{2}k = \tfrac{1}{2}\sqrt{(4r^2-h^2)} - \frac{hk}{2\sqrt{(4r^2-k^2)}} : \frac{k\sqrt{(4r^2-h^2)}}{4r} - \frac{hk^2}{4r\sqrt{(4r^2-k^2)}}$$

$$DI + IF = DF = \frac{rh}{\sqrt{(4r^2-k^2)}} + \frac{k\sqrt{(4r^2-h^2)}}{4r} - \frac{hk^2}{4r\sqrt{(4r^2-k^2)}}$$

$$DF = \frac{(4r^2-k^2)h + k\sqrt{(4r^2-h^2)(4r^2-k^2)}}{4r\sqrt{(4r^2-k^2)}}$$

$$DF = \frac{h\sqrt{(4r^2-k^2)} + k\sqrt{(4r^2-h^2)}}{4r}$$

$$2DF = c = \frac{h\sqrt{(4r^2-k^2)} + k\sqrt{(4r^2-h^2)}}{2r}$$

§. 175.

Ex §§. 163. 164.

sit $\tfrac{1}{2}h = \text{sin.}\, x$, & $\sqrt{(r^2-\tfrac{1}{4}h^2)} = \text{cosin.}\, x$

$\tfrac{1}{2}k = \text{sin.}\, y$ & $\sqrt{(r^2-\tfrac{1}{4}k^2)} = \text{cosin.}\, y$

erit

$$DF = \text{sin.}\,(x+y) = \frac{\tfrac{1}{2}h\sqrt{(r^2-\tfrac{1}{4}k^2)} + \tfrac{1}{2}k\sqrt{(r^2-\tfrac{1}{4}h^2)}}{r}$$

$$\text{sin.}\,(x+y) = \text{sin.}\, x \,.\, \text{cosin.}\, y + \text{sin.}\, y\ \text{cos.}\, x$$

§. 201.

Elegans occurrit *reductionis* specimen

$$y^2 + (ax+b)\,y + cx^2 + ex + f = y^2 + (\alpha x + \beta)\,y + \gamma x^2 + \varepsilon x + \varphi$$

& fiet operatione ipsa instituta

$$\left.\begin{array}{l}(\gamma-c)^2\\+a(a-\alpha)(\gamma-c)\\+c(a-\alpha)^2\end{array}\right\}x^4\;\left.\begin{array}{l}+2(\gamma-c)(\varepsilon-e)\\+a(a-\alpha)(\varepsilon-e)\\+(a-\alpha)b(\gamma-c)\\+a(b-\beta)(\gamma-c)\\+(a-\alpha)^2e\\+2(a-\alpha)(b-\beta)c\end{array}\right\}x^3\;\left.\begin{array}{l}+(\varepsilon-e)^2\\+2(\gamma-c)(\varphi-f)\\+a(a-\alpha)(\varphi-f)\\+(a-\alpha)b(\varepsilon-e)\\+a(b-\beta)(\varepsilon-e)\\+(b-\beta)b(\gamma-c)\\+(a-\alpha)^2f\\+2(a-\alpha)(b-\beta)e\\+(b-\beta)^2c\end{array}\right\}x^2\;\left.\begin{array}{l}+2(\varepsilon-e)(\varphi-f)\\+(a-\alpha)b(\varphi-f)\\+a(b-\beta)(\varphi-f)\\+(b-\beta)b(\varepsilon-e)\\+2(a-\alpha)(b-\beta)f\\+(b-\beta)^2e\end{array}\right\}x\;\begin{array}{l}+(\varphi-f)^2\\+(b-\beta)b(\varphi-f)=0\\+(b-\beta)^2f\end{array}$$

§. 262.

$$\sqrt{(2+\sqrt{-3})}+\sqrt{(2-\sqrt{-3})}=\sqrt{(4+2\sqrt{7})}$$

Dem.

Sit $\sqrt{-3}=x$, erit
$\sqrt{(2+x)}+\sqrt{(2-x)}=$ quantitati ſummandæ
ponatur $\sqrt{(2+x)}=y$, & $\sqrt{(2-x)}=z$

ergo

$2+x=y^2$	$\sqrt{(2-x)}=z$
$2-x=z^2$	$\sqrt{(2+x)}=y$
$4=y^2+z^2$	$\sqrt{(4-x^2)}=zy$
$4+2\sqrt{7}=y^2+2yz+z^2$	$2\sqrt{(4+3)}=2zy$

$\sqrt{(4+2\sqrt{7})}=y+z$, ergo ſumma iſtarum quantitatum imaginariarum eſt realis, & productum $1+\sqrt{(2+\sqrt{3})}\times 1+\sqrt{(2-\sqrt{-3})}=1+\sqrt{(4+2\sqrt{7})}+\sqrt{7}$ pariter reale:

§. 310.

Sit x^m, & x abeat in $x+e$, erit

$$(x+e)^m = x^m + \frac{mex^{m-1}}{1} + \frac{m\,.\,m-1}{1\,.\,2}\,e^2x^{m-2}$$

Si prior series ab hac subtrahatur, habebitur

$$(x+e)^m - x^m = \frac{mex^{m-1}}{1} + \frac{m.m-1}{1.2}\,e^2x^{m-2} + \frac{m.m-1.m-2}{1.2.3}\,e^3x^{m-3}$$

abeat iterum x in $(x+e)$ erit

$$(x+e)^{m-1} = x^{m-1} + \frac{m-1.ex^{m-2}}{1} + \frac{m-1.m-2}{1.2.}\,e^2x^{m-3}$$

$$(x+e)^{m-2} = x^{m-2} + \frac{m-2}{1}\,ex^{m-3} + \frac{m-2.m-3}{1.2}\,e^2x^{m-4}$$

$$(x+e)^{m-3} = x^{m-3} + \frac{m-3}{1}\,ex^{m-4} + \frac{m-3.m-4}{1.2}\,e^2x^{m-5}$$

&c. &c. &c.

$$(me)(x+e)^{m-1} = mex^{m-1} + \frac{m.m-1.e^2x^{m-2}}{1} + \frac{m.m-1.m-2}{1.2}\,e^3x^{m-3}$$

$$\frac{m.m-1}{1.2}\,e^2(x+e)^{m-2} = \frac{m.m-1.}{1.2}\,e^2x^{m-2} + \frac{m.m-1.m-2}{1.2}\,e^3x^{m-3}$$

$$\frac{m.m-1.m-2}{1.2.3}\,e^3(x+e)^{m-3} = \frac{m.m-1.m-2}{1.2.3}\,e^3x^{m-3} + \frac{m.m-1.m-2.m-3}{1.2.3}\,e^4x^{m-4}$$

$$\frac{m.m-1.m-2.m-3}{1.2.3.4}\,e^4(x+e)^{m-4} = \frac{m.m-1.m-2.m-3}{1.2.3.4}\,e^4x^{m-4}$$

Subtrahatur hæc series à prima differentiali, erit

$$+\frac{m.m-1.m-2}{1.2.3}e^{3}x^{m-3}\quad+\frac{m.m-1.m-2.m-3}{1.2.3.4}e^{4}x^{m-4}$$

$$+\frac{m.m-1.m-2.m-3}{1.2.3.4}e^{4}x^{m-4}\quad+=1^{ma}$$ ſeries differ.

$$+\frac{m-1.m-2.m-3}{1.2.3}e^{3}x^{m-4}\quad+\frac{m-1.m-2.m-3.m-4}{1.2.3.4}e^{4}x^{m-5}$$

$$+\frac{m-2.m-3.m-4}{1.2.3}e^{3}x^{m-5}\quad+\frac{m-2.m-3.m-4.m-5}{1.2.3.4}e^{4}x^{m-6}$$

$$+\frac{m-3.m-4.m-5}{1.2.3}e^{3}x^{m-6}\quad+\frac{m-3.m-4.m-5.m-6}{1.2.3.4}e^{4}x^{m-7}$$

$$+\frac{m.m-1.m-2.m-3}{1.2.3}e^{4}x^{m-4}$$

$$+\frac{m.m-1.m-2.m-3}{1.2.2}e^{4}x^{m-4}\quad+\frac{m.m-1.m-2.m-3.m-4}{1.2.2.3}e^{5}x^{m-5}$$

$$+\frac{m.m-1.m-2.m-3.m-4}{1.2.2.3}e^{5}x^{m-5}\quad+\frac{m.m-1.m-2.m-3.m-4.m-5}{1.2.2.3.3}e^{6}x^{m-6}$$

$$\frac{m.m-1.e^2 x^{m-2}}{1} + \frac{m.m-1.m-2.e^3 x^{m-3}}{1.1}$$

abeat denuo

$$m.m-1.e^2(x+e)^{m-2} = m.m-1.e^2 x^{m-2} + \frac{m.m-1.m-2}{1} e^3 x^{m-3}$$

$$(m.m-1.m-2)\, e^3 (x+e)^{m-3} = \frac{m.m-1.m-2}{1} e^3 x^{m-3}$$

$$\frac{m.m-1.m-2.m-3}{1.3} e^4 (x+e)^{m-4} =$$

$$\frac{m.m-1.m-2.m-3}{2.2} e^4 (x+e)^{m-4} =$$

$$m.m-1.m-2\, e^3 x^{m-3} \quad \left.\begin{array}{l} +\frac{m.m-1.m-2.m-3}{1.1} \\ +\frac{m.m-1.m-2.m-3}{1.2} \end{array}\right\} e^4 x^{m-4}$$

fiat $x = x + e$

$$m.m-1.m-2\, e^3 (x+e)^{m-3} = m.-1.m-2\, e^3 x^{m-3}$$

$$m.m-1.m-2.m-3\, e^4 (x+e^{m-4} =$$

$$\frac{m.m-1.m-2.m-3}{1.2} e^4 (x+e)^{m-4} =$$

$$m.m-1.m-2.m-3\, e^4 x^{m-4}$$

$$\left.\begin{array}{l} + \dfrac{m\,.\,m-1\,.\,m-2\,.\,m-3}{1\,.\,3} \\ + \dfrac{m\,.\,m-1\,.\,m-2\,.\,m-3}{2\,.\,2} \end{array}\right\} e^{4}\,x^{m-4} \quad + = 2^{da} \text{ series differentialis}$$

x in $x + e$

$$+ \frac{m\,.\,m-1\,.\,m-2\,.\,m-3\,.}{1\,.\,2}\, e^{4}\, x^{m-4} \qquad + \frac{m.m-1.m-2.m-3.m-4}{1.2.3}\, e^{5}\, x^{m-5}$$

$$+ m\,.\,m-1\,.\,m-2\,.\,m-3\; e^{4}\, x^{m-4} \qquad + \frac{m\,.\,m-1.m-2.m-3.m-4}{1\,.\,2}\, e^{5}\, x^{m-5}$$

$$\frac{m\,.\,m-1\,.\,m-2\,.\,m-3}{1\,.\,3}\, e^{4}\, x^{m-4} \qquad + \frac{m.m-1\,.\,m-2.\,m-3.m-4}{1\,.\,3}\, e^{5}\, x^{m-5}$$

$$\frac{m\,.\,m-1\,.\,m-2\,.\,m-3}{2\,.\,2}\, e^{4}\, x^{m-4} \qquad + \frac{m.m-1\,.\,m-2\,.\,m-3.m-4}{2\,.\,2}\, e^{5}\, x^{m-5}$$

= series 3^{tia} differentialis.

$$+ m\,.\,m-1\,.\,m-2\,.\,m-3\; e^{4}\, x^{m-4}$$

$$m\,.\,m-1\,.\,m-2\,.\,m-3\; e^{4}\, x^{m-4}$$

$$\frac{m\,.\,m-1\,.\,m-2\,.\,m-3}{1\,.\,2}\, e^{4}\, x^{m-4}$$

+ &c. = series 4^{ta} differentialis.

Applicemus hanc theoriam ad numeros ſequentes

Diff. 2.	Diff. 2.	Quadrat.	Radices.
		3095920881	55641
2	111283	3096032164	55642
2	111285	3096143449	55643
2	111287	3096254736	55644
	111289	3096366025	55645

Differ. 2.		Differ. 1.	Quadrat.	Radices
			3095920881	55641
	8	222568	3096143449	55643
$2 . 2^2 =$	8	222576	3096366025	55645
	8	222584	3096588609	55647
		222592	3096811201	55649

Differ. 2.		Differ. 1.	Quadrat.	Radices
			3095920881	55641
	18	333855	3096254736	55644
$2 . 3^2 =$	18	333873	3096588609	55647
	18	333891	3096922500	55650
		333909	3097256409	58653

Differ. 2.		Differ. 1.	Quadrat.	Radices
			3095920881	55641
	50	556435	3096477316	55646
$2 . 5^2 =$	50	556485	3097033801	55651
	50	556535	3097590336	55656
		556585	3098146921	55661

Diff,

Diff. 4æ	Diff. 3æ	Diff. 2æ	Diff. 1æ	Biquadrat.	Radices
				16	2
			65	81	3
		110	175	256	4
1 . 3 24	48	194	369	625	5
$1.2.3.4 = 24$	108	302	671	1296	6
24	132	434	1105	2401	7
	156	590	1695	4096	8

Diff. 4.	Diff. 3.	Diff. 2.	Diff. 1.	Biquadrat	Radices
				16	2
			240	256	4
		800	1040	1296	6
384	960	1760	2800	4096	8
$24 . 2^4 = 384$	1344	3104	5905	10000	10
384	1728	4832	10736	20736	12
	2112	6944	17680	38416	14

Diff. 4.	Diff. 3.	Diff. 2.	Diff. 1.	Biquadrat	Radices.
				16	2
			609	625	5
		2862	3471	4096	8
1944	4212	7074	10545	14641	11
$24 . 3^4 = 1944$	6156	13230	23775	38416	14
1944	8100	21330	45105	83521	17
	10044	31374	76479	160000	20

Diff. 4.	Diff. 3.	Diff. 2.	Diff. 1.	Biquadrat	Radices
				16	2
			2385	2401	7
		15950	18335	20736	12
15000	28500	44450	62785	83521	17
$24 . 5^4 = 15000$	43500	87950	150735	234256	22
15000	58500	146450	297185	53144	27
	73500	219950	517135	1048576	32

Diff. 3.	Diff. 2.	Diff. 1.	Cubi	Radices
			66923416	406
	2442	495727	67419143	407
6	2448	498169	67917312	408
$1.2.3 = 6$	2454	500617	68417929	409
6	2460	503071	68921000	410
		505531	69426531	411

Diff. 3.	Diff. 2.	Diff. 1.	Cubi	Radices
			66923416	406
		993896	67917312	408
48	9792	1003688	68921000	410
$6.2^3 = 48$	9840	1013528	69934528	412
48	9888	1023416	70957944	414
	9936	1033352	71991296	416

Diff. 2.		Diff. 1.	Cubi	Radices
			66923416	406
	22086	1494513	68417929	409
162	22248	1516599	69934528	412
$6.3^3 = 162$	22410	1538847	71473375	415
162	22572	1561257	73034632	418
		1583829	74618461	421

			Cubi	Radices
			66923416	406
	39360	1997584	68921000	410
$6.4^3 = 384$	39744	2036944	70957944	414
384	40128	2076688	73034632	418
		2116816	75151448	422

§§ 80

§§. 80. & seqq. aliquot problemata utilia adjiciam.

Probl.

Datis quatuor numerorum in progressione arithmetica summa a, & cuborum eorundem summa b, invenire ipsos numeros.

Sol.

Sit x terminus primus, & y secundus, & erunt termini x; y; $\frac{1}{2}a - y$; $\frac{1}{2}a - x$. & ex natura progressionis arithmeticæ est $2a = \frac{1}{2}y - y + x$, ergo $3y = \frac{1}{2}a + x$ hinc progressio fiet x; $\frac{a+2x}{6}$; $\frac{a-x}{3}$; $\frac{a-2x}{2}$ vel x; $\frac{1}{6}a + \frac{1}{3}x$; $\frac{1}{3}a - \frac{1}{3}x$; $\frac{1}{2}a - x$. Jam summa cuborum horum terminorum debet esse $= b$.

$$x^3$$
$$\tfrac{1}{216}a^3 + \tfrac{1}{36}a^2x + \tfrac{1}{18}ax^2 + \tfrac{1}{27}x^3$$
$$\tfrac{1}{27}a^3 - \tfrac{1}{9}a^2x + \tfrac{1}{9}ax^2 - \tfrac{1}{27}x^3$$
$$\tfrac{1}{8}a^3 - \tfrac{3}{4}a^2x - \tfrac{3}{4}ax^2 - x^3$$
$$\tfrac{5}{3}ax^2 - \tfrac{5}{6}a^2x + \tfrac{1}{6}a^3 = b$$
$$x^2 - \tfrac{1}{2}ax + \tfrac{1}{10}a^2 = \frac{3b}{5a}$$
$$x = \tfrac{1}{4}a \pm \sqrt{\left(\frac{48b - 3a^3}{80a}\right)}$$

Probl.

Cognitis progressionis arithmeticæ summa terminorum a & summa cuborum eorundem b, & numero terminorum c, invenire ipsos numeros

Sol.

In serie arithmetica sequente, ubi A est terminus primus & d differentia

$$A;\ A+d;\ A+2d;\ A+3d;\ A+4d;\ A+5d;\ A+6d = a$$
$$d + 2d + 3d + 4d + 5d + 6d = a - 7A$$
$$d(1 + 2 + 3 + 4 + 5 + 6) = a - 7A$$

Cum

Cum data sit summa progressionis arithmeticæ $= a$, & numerus terminorum sit $= c$, vocemus $c - 1 = n$, & primum terminum $= x$

$$\frac{a - cx}{1.2.3\ldots + n} \quad \text{vel} \quad \frac{a - (n+1)x}{1+2+3+\ldots n} = d$$

jam vero $1 + 2 + 3 + 4 + \ldots\ldots n = n + 1 . \frac{n}{2}$ §. 90.

hinc formula pro invenienda differentia d abit in sequentem $\frac{a-(nx+1)x}{(nn+n):2}$

$$= \frac{2a}{n^2 + n} - \frac{2x}{n}, \text{ ergo progressio habebit formam sequentem}$$

$$x;\ \left(\frac{n-2}{n}\right)x + \frac{2a}{n^2+n};\ \left(\frac{n-4}{n}\right)x + \frac{4a}{n^2+n};\ \left(\frac{n-6}{n}\right)x + \frac{6a}{n^2+n};\ \left(\frac{n-8}{n}\right)x + \frac{8a}{n^2+n}$$

Jam ex forma apparet, seriem esse arithmeticam, quia quilibet terminus bis sumtus æquivalet suis duobus utrinque positis vicinis v. g.

$$\left(\frac{(n-4)x}{n} + \frac{4a}{n^2+n}\right) \times 2 = \left(\frac{n-2}{n}\right)x + \frac{2a}{n^2+n} + \left(\frac{n-6}{n}\right)x + \frac{6a}{n^2+n}.$$

Deinde quicunque sit terminorum numerus, summa semper est $= a$, namque posito $c = 4$ adeoque $n = 3$, quatuor termini faciunt summam a quia terminorum æqualiter ab extremis distantium membra x continentia sese destruunt, inde etiam sequitur si ad cubos horum terminorum conficiendos progrediamur, summam omnium ubi x^3 evanescere,

Cubi.

$$x^3$$

$$\left(\frac{n-2}{n}\right)^3 x^3 + \frac{3(n-2)^2\, 2ax^2}{n^2(n^2+n)} + \frac{3(n-2)4a^2x}{n(n^2+n)^2} + \frac{8a^3}{(n^2+n)^3}$$

$$\left(\frac{n-4}{n}\right)^3 x^3 + \frac{3(n-4)^2\, 4ax^2}{n^2(n^2+u)} + \frac{3(n-4)16a^2x}{n(n^2+n)^2} + \frac{64a^3}{(n^2+n)^3}$$

Jam in summandis his terminis negligamus eos, ubi x^3 occurrit, & summa cuborum sequentem æquationem producet.

$$\frac{2(n-2)^2 + 4(n-4)^2 + 6(n-6)^2 + 8(n-8)^2 + 10(n-10)^2 \text{ \&c. in } 3ax^2}{n^2\ (n^2+n)} +$$

$$\frac{4(n-2) + 16(n-4) + 36(n-6) + 64(n-8) + 81(n-10) \text{ \&c. in } 3a^2x}{n(n^2+n)^2}$$

$$\frac{+\ 8 + 64 + 216 + 512 \text{ \&c. in } a^3}{(n^2+n)^3} = b$$

Quæ æquatio cum ſit quadratica, facile ex ea invenietur valor primi termini x

ſcilicet

$$x^2 + \frac{4(n-2) + 16(n-4) + 36(n-6) + 64(n-8) + \text{\&c. in } ax}{2(n-2)^2 + 4(n-4)^2 + 6(n-6)^2 + \text{\&c. in } (n+1)} =$$

$$\frac{(8+64+216+512+\text{\&c.})\text{in} - \frac{1}{3}a^2}{2(n-2)^2+4(n-4)^2+6(n-6)^2+\text{\&c. in }(n+1)^2}$$

$$+ \frac{b\,(n^2\,.\,n^2+n)}{2(n-2)^2+4(n-4)^2+6(n-6)^2 + \text{in } 3a}$$

Exempl.

Quæruntur ſex numeri in progreſſione arithmetica, quorum ſumma ſit 21, ſumma cuborum 441. Hic itaque $a = 21$, & $b = 441$; $c = 6$, unde $n = 5$. hinc primum membrum rationale

$$\frac{(4.3+16.1+36.-1+64.-3+100.-5) \text{ in } -\frac{21}{2}}{(2.9+4.1+6.1+8.9+10.25) \text{ in } 6} \text{ vel}$$

$$\frac{(12+16-36-192-500) \text{ in } -\frac{21}{2}}{(18+4+6+72+250) \text{ in } 6} = \frac{-700 \times -\frac{21}{2}}{350 \times 6} \text{ vel } \frac{7}{2}$$

unde $(\frac{7}{2})^2 = \frac{49}{4}$ conficiet primum membrorum, quæ ſub vinculo radicali ſunt.

Quoad

Quoad alterum membrum, quod ſub vinculo eſt, habemus

$$\frac{(8+64+216+512+1000)\times -\frac{441}{3}}{(2.9+4.1+6.1+8.9+10.25)\times 36} = \frac{-264600}{12600} = -21$$

Tertium membrum quod ſub vinculo eſt

$$\frac{441\times 25\times 30}{(18+4+6+72+250)\times 63} = \frac{330750}{350\times 63} = 15.$$

hinc totum membrum irrationale erit $\sqrt{(\frac{49}{4}-21+15)} = \frac{5}{2}$

Ergo terminus extremus major $\frac{7}{2}+\frac{5}{2}=6$ & minor $\frac{7}{2}-\frac{5}{2}=1$ differentia 5 diviſa per $n=5$ eſt differentia progreſſionis unde numeri 1.2.3.4.5.6

2. Exempl.

Quæruntur numeri novem in progreſſione arithmetica quorum ſumma $=39$ & ſumma cuborum $=927\frac{1}{3}$ ergo $a=39$, $b=927\frac{1}{3}$; $n=8$

Membrum rationale erit

$$\frac{(4.6+16.4+36.2+64.0+100.-2+144.-4+196.-6+256.-8)\times -\frac{39}{2}}{(2.36+4.16+6.4+8.0+10.4+12.16+14.36+16.64)\times 9}$$

quod valet $\dfrac{-3840\times -\frac{39}{2}}{(72+64+24+40+192+504+1024)\times 9} = \dfrac{74880}{17280} = 4\frac{1}{3}$

unde $(4\frac{1}{3})^2 = \frac{169}{9}$ eſt primum membrum ſub vinculo.

Secundum membrum ſub vinculo fiet

$$\frac{(8+64+216+512+1000+1728+2744+4096)\times -507}{1920\times 81} = \frac{-5256516}{155520} = -33\frac{4}{5}$$

Tertium ſub vinculo membrum

$\dfrac{64\times 927\frac{1}{3}\times 72}{1920\times 117} = 19\frac{1}{45}$ Hinc erit integrum membrum irrationale

$\sqrt{(\frac{169}{9}-33\frac{4}{5}+19\frac{1}{45})} = 2$

Vnde

Vnde extremi progressionis termini erunt major $= 4\frac{1}{3} + 2$; minor $= 4\frac{1}{3} - 2$, horum ergo differentia 4 dividatur per 8, quotiens $\frac{1}{2}$ erit differentia progressionis, adeoque numeri

$$\frac{14}{6};\ \frac{17}{6};\ \frac{20}{6};\ \frac{23}{6};\ \frac{26}{6};\ \frac{29}{6};\ \frac{32}{6};\ \frac{35}{6};\ \frac{38}{6}$$

§. 105.

Invenire quinque numeros in progressione geometrica, quorum summa sit a & summa quadratorum = b

Sol.

Sint numeri x; xy; xy^2; xy^3; xy^4, erit itaque

$$a = x(1 + y + y^2 + y^3 + y^4) = \frac{x(1 - y^5)}{1 - y}$$

$$b = x^2(1 + y^2 + y^4 + y^6 + y^8) = \frac{xx(1 - y^{10})}{1 - yy}$$ unde

$$\frac{b}{aa} = \frac{(1 + y^5)(1 - y)}{(1 - y^5)(1 + y)}$$ fit $\frac{b}{aa} = \frac{m}{n}$, & $y = \frac{1 - z}{1 + z}$

hinc $\frac{m}{n} = \frac{2z}{2} \cdot \frac{2 + 20z^2 + 10z^4}{10z + 20z^3 + 2z^5} = \frac{1 + 10z^2 + 5z^4}{5 + 10z^2 + z^4}$, ergo

$$mz^4 + 10mz^2 + 5m = 5nz^4 + 10nz^2 + n$$ seu

$$z^4 = \frac{10(m - n)z^2 + 5m - n}{5n - m}$$

$$z^2 = \frac{5(m - n) \pm 2\sqrt{(5m^2 - 6mn + 5n^2)}}{5n - m}$$

§. 64.

Sit insula datæ perimetri, sintque duo viatores eodem tempore ex eodem perimetri loco A versus eandem plagam egredientes, & insulam datis celeritatibus

 cir-

circumeuntes, ſitque celeritas præcedentis major celeritate ſequentis: determinare locum concurſus B, numerum dierum itineris, itemque quoties priori, quoties poſteriori inſula circumeunda ſit, donec ſe invicem aſſequantur.

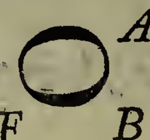
Solutio.

Sit celeritas tardioris $= c$; Perimeter inſulæ $= p$

— — — celerioris $= C$; numerus dierum itineris $= x$

totum iter tardioris $= v$

Quoniam celeritates dantur per ſpatia eodem tempore confecta, adeoque & per itinera diurna exhiberi poſſunt, differentia itinerum diurnorum exprimatur per $C - c$. Porro quia tardior ſequens celeriorem præeuntem nunquam aſſequitur, ex natura problematis, nempe per conditionem viæ in ſe redeuntis intelligitur, quod præcedens ſuæ celeritatis exceſſu tardiorem à tergo tandem ſit aſſecuturus, ſimul ac exceſſus celeritatum ſeu itinerum diurnorum aliquoties repetitus totam inſulæ perimetrum adæquaverit, eo enim *ABFA* intervallo



ſub initium motus celerior à tardiore diſtare concipiendus eſt. Quoniam igitur celeritate manente eadem ſpatia ſunt temporibus proportionalia; erit *differentia itinerum diurnorum i. e. ſpatium, quod celerior præ tardiore uno die lucratur, ad ſpatium quod toto tempore quæſito lucrandum eſt, ut tempus unius diei ad tempus quæſitum, quo celerior tardiorem aſſequitur*

eſt adeo $C - c : p = 1 : x$ vel

$$x = \frac{p}{C-c}$$

Cum præterea ſit *tempus unius diei ad totum tempus quæſitum ut iter diurnum tardioris ad totum iter á tardiore in tempore quæſito conficiendum*, erit

$$1 : x = c : v$$

$$1 : \frac{p}{C-c} = c : v$$

$$v = \frac{cp}{C-c}$$

In qua expreſſione cum p denotet perimetrum inſulæ ſeu unam circuitionem, ejus coefficiens $\frac{c}{C-c}$ indicabit numerum circuitionum à tardiore factarum, cui ſi addatur unitas, prodibit $\frac{c}{C-c} + 1 = \frac{C}{C-c} =$ numero circuituum à celeriori factorum. Tandem ſi in caſibus ſpecialibus $\frac{c}{C-c}$ æquivalet numero integro, manifeſtum eſt, tum locum concurſus contingere in ipſo loco egreſſus A, ſeu eſſe diſtantiam $AB = 0$, ſi vero $\frac{c}{C-c}$ eſt numerus fractus, diviſione actu inſtituta, fractio reſidua exprimet rationem, quam habet ultima circuitio non abſoluta ad unam circuitionem integram, i. e. dabit rationem quam habet loci concurſus B à loco egreſſus A diſtantia AB ad integram perimetrum $ABFA$.

Sit p 30 milliar. $C = 15$; $c = 10$, erit $x = 6$, & numerus circuitionum à tardiore factarum $= 2$. Cadit ergo B in A, & $AB = 0$.

Sit $p = 30$; $C = 16$; $c = 10$; erit $x = 5$, & numerus circuitionum à tardiore factarum $= 1\frac{2}{3}$, hinc $AB = 20$ milliar.

§. 190.

Probl.

Invenire duos numeros x & y hujus naturæ ut $x^y = y^x$

Solut.

Ponatur $az = x$ & $y = bz$, & erit

$$(az)^{bz} = (bz)^{az}$$

$$a^b z^b = b^a z^a$$

$$a^b = b^a z^{a-b}; \quad \frac{a^b}{b^a} = z^{a-b}; \quad z = \frac{a^{b:a-b}}{b^{a:a-b}}$$

ſi

sit $a - b = m$, & erit $z = \dfrac{a^{a-m:m}}{(a-m)^{a:m}}$

si $m = 1$, erit $z = \dfrac{a^{a-1}}{(a-1)^a} = \dfrac{a^a}{a(a-1)^a}$

adeoque $az = \left(\dfrac{a}{a-1}\right)^a$ & $bz = \left(\dfrac{a}{a-1}\right)^{a-1}$

sit $a - 1 = n$, & hæ duæ quantitates az & bz erunt $\left(1 + \dfrac{1}{n}\right)^n$ & $\left(1 + \dfrac{1}{n}\right)^{1+n}$

Exempl.

sit $n = \frac{1}{2}$, erit $x = \sqrt{3}$ & $y = 3^{3:2}$

ergo debet esse

$(\sqrt{3})^{\sqrt{27}} = (\sqrt{27})^{\sqrt{3}}$, & progrediendo ad logarithmos $(\sqrt{27})\ \frac{1}{2}\ l\ 3$

$= (\sqrt{3})\ \frac{1}{2}\ l\ 27$

$(\sqrt{27})\ \frac{1}{2}\ l\ 3 = (\sqrt{3})\ \frac{3}{2}\ l\ 3$

$\sqrt{27} = 3\sqrt{3}$

sit $n = 3$, & erit $x = (\frac{4}{3})^3$ & $y = (\frac{4}{3})^4$

vel $x = \frac{64}{27}$ & $y = \frac{256}{81}$ ergo

$(\frac{64}{27})^{256:81} = (\frac{256}{81})^{64:27}$

$\frac{256}{81}\ (3l4 - 3l3) = \frac{64}{27}\ (4l4 - 4l3)$

$\frac{3 \cdot 256}{81}\ (l4 - l3) = \frac{4 \cdot 64}{27}\ (l4 - l3)$

$\frac{3 \cdot 256}{81} = \frac{4 \cdot 64}{27}$

fit $n = 2$, erit $x = (\frac{3}{2})^2$ & $y = (\frac{3}{2})^3$
vel $x = \frac{9}{4}$ & $y = \frac{27}{8}$, ergo $(\frac{9}{4})^{27:8} = (\frac{27}{8})^{9:4}$ vel $(\frac{9}{4})^{27} = (\frac{27}{8})^{18}$
& $27\,(2l3 - 2l2) = 18\,(3l3 - 3l2)$ vel $2 \cdot 27 = 3 \cdot 18$.
Sit $x = 2$, & $y = 4$ erit $2^4 = 4^2$

§. 725.

$$s = ax - bx^2 + cx^3 - dx^4 + ex^5 - \&c.$$

ponatur $x = \frac{y}{1-y} = y + y^2 + y^3 + y^4 + y^5 + \&c.$

$$\begin{array}{l} s = ay + ay^2 + ay^3 + ay^4 + ay^5 \\ \qquad\qquad - by^2 - 2by^3 - 3by^4 \\ \qquad\qquad\qquad + cy^3 + 3cy^4 \\ \qquad\qquad\qquad\qquad - dy^4 \end{array}$$

$$s = ay - (b-a)y^2 + (c-2b+a)y^3 - (d-3c+3b-a)y^4 \ \&c.$$

fit $x = 1$ & $y = \frac{1}{2}$, erit

$$s = a - b + c - d + e - f \ \&c.$$

$$s = \frac{a}{2} - \frac{(b-a)}{4} + \frac{c-2b+a}{8} - \frac{(d-3c+3b-a)}{16}$$

Ex.

$$\begin{array}{l} 1 - 4 + 9 - 16 + 25 - 36 \ \&c. \\ \quad 3 \quad 5 \quad 7 \quad 9 \\ \qquad 2 \quad 2 \quad 2 \end{array}$$

Ergo $s = \frac{1}{2} - \frac{3}{4} + \frac{2}{8} = 0$

$$\begin{array}{l} 1 - 2 + 3 - 4 + 5 - 6 \\ \quad 1 \quad 1 \end{array}$$

ergo $s = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

§. 13.

§. 13

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 \text{ \&c.}$$

$$\frac{1}{1-x} = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16}) + \text{ in infin.}$$

$$1 = (1-x^2)(1+x^2)(1+x^4)(1+x^8)(1+x^{16}) \text{ \&c. in infin.}$$

$$1 = 1 - x^{32} \text{ \&c. quia } x < 1.$$

§. 690.

$$z = ay + by^2 + cy^3 + dy^4 + ey^5 \text{ \&c.}$$

quæritur expressio $\tau\varepsilon$ y per z

Assumatur series

$$y = Az + Bz^2 + Cz^3 + Dz^4 + Ez^5 \text{ \&c.}$$

& determinentur coefficientes A, B, C, D, E &c.

erit

$$\begin{array}{lllllll}
ay = & aAz & + aBz^2 & + aCz^3 & + aDz^4 & + aEz^5 & \text{\&c.} \\
by^2 = & & bA^2z^2 & + 2bABz^3 & + bB^2z^4 & + 2bBCz^5 & \text{\&c.} \\
 & & & & + 2bACz^4 & & \\
cy^3 = & & & cA^3z^3 & + 3cA^2Bz^4 & + 3cB^2Az^5 & \text{\&c.} \\
dy^4 = & & & & + dA^4z^4 & + 4dA^3Bz^5 & \text{\&c.} \\
ey^5 = & & & & & + eA^5z^5 & \text{\&c.} \\
\text{\&c.} & & & & & & \\
-z = -z & & & & & &
\end{array}$$

$$0 = \left.\begin{array}{l} aA \\ -1 \end{array}\right\} z \left.\begin{array}{l} +aB \\ +bA^2 \end{array}\right\} z^2 \left.\begin{array}{l} +aC \\ +2bAB \\ +cA^3 \end{array}\right\} z^3 \left.\begin{array}{l} +aD \\ +bB^2 \\ +2bAC \\ +3cA^2B \\ +dA^4 \end{array}\right\} z^4 \left.\begin{array}{l} +aE \\ +2bBC \\ +3cB^2A \\ +4dA^3B \\ +eA^5 \end{array}\right\} z^5$$

Ja

Jam est $aA - 1 = 0$

$$aB + bA^2 = 0$$

$$aC + 2bAB + cA^3 = 0$$

unde oriuntur valores coefficientium ABC & in serie assumta

$$A = \frac{1}{a}$$

$$B = -\frac{bA^2}{a} = -\frac{b}{a^3}$$

$$C = -\frac{2bAB - cA^3}{a} = \frac{2b^2}{a^5} - \frac{c}{a^4}$$

$$D = -\frac{bB^2 - 2bAC - 3A^2B - dA^4}{a}$$

Si proposita fuerit series sequens

$z = ay + cy^3 + ey^5 + gy^7 + iy^9$ &c. erit

$$y = \frac{z}{a}$$

$$-\frac{cA^3z^3}{a}$$

$$-\frac{3cA^2C - eA^5}{a}z^5$$

$$-\frac{3cC^2A - 3cEA^2 - 5eCA^4 - gA^7}{a}z^7$$

Res inde manifesta est, si in expressione antecedente omittantur quantitates, quas ingrediuntur bdf &c. tunc enim prodit hæc series v. g. $aC + 2bAB + cA^3 = 0$, quoniam secundus terminus b affectus est, negligatur ut sit $aC + cA^3 = 0$, erit $C = -\frac{cA^3}{a}$

Si denique proponatur ſequens ſeries

$$az + bz^2 + cz^3 + dz^4 \text{ \&c.} = gy + hy^2 + iy^3 + ky^4 + ly^5 \text{ \&c.}$$

& quæratur expreſſio τȣ z per y

$$z = \frac{g}{a} \cdot y$$
$$+ \frac{h - bA^2}{a} y^2$$
$$+ \frac{i - 2bAB - cA^3}{a} y^3$$
$$+ \frac{k - bBB - 2bAC - 3cA^2B - dA^4}{a} y^4$$
$$+ \frac{l - 2BC - 2bAD - 3cABB - 3cA^2C - 4dA^3B - eA5}{a} y^5$$

Sit enim $z = Ay + By^2 + Cy^3 + Dy^4 + Ey^5$ &c. erit

$$\begin{array}{llllll}
az = & aAy & + aBy^2 & + aCy^3 & + aDy^4 & + aEy^5 \\
bz^2 = & & + bA^2y^2 & + 2bABy^3 & + bB^2y^4 & + 2bBCy^5 \\
 & & & & + 2bACy^4 & \\
cz^3 = & & & cA^3y^3 & + 3cA^2By^4 & + 3cB^2Ay^5 \\
dz^4 = & & & & dA^4y^4 & + 4dA^3By^5 \\
ez^5 = & & & & & eA^5y^5 \\
\text{\&c.} & & & & & \\
gy = & gy & & & & \\
hy^2 = & & + hy^2 & & & \\
iy^3 = & & & + iy^3 & & \\
ky^4 = & & & & + ky^4 & \\
ly^5 = & & & & & + ly^5
\end{array}$$

Ergo $gy = aAy$ & $A = \frac{g}{a}$

$hy^2 = bA^2y^2 + aBy^2$ hinc $B = \frac{h - bA^2}{a}$

$iy^3 = cA^3y^3 + 2bABy^3 + aCy^3$, hinc $C = \frac{i - cA^3 - 2bAB}{a}$

§. 294.

§. 294.

Methodus Newtoniana inveniendi divisores æquationum.

Proponatur æquatio $x^4 + ax^3 + bx^2 + cx + d = 0$ quæritur an habeat divisores.

Ponamus esse divisorem $x + \alpha = 0$, ita ut formula sit

$$x^4 + ax^3 + bx^2 + cx + d = (x + \alpha)(x^3 + fx^2 + gx + h)$$

ergo hæc formula est divisibilis per $x + \alpha$, quicquid ponatur pro x, ergo posito $x = 0$; d erit divisibiie per α

si ponatur $x = 1$; erit $1 + a + b + c + d$ divisibile per $1 + \alpha$

si $x = 2$; erit $16 + 8a + 4b + 2c + d$ divisibile per $2 + \alpha$
&c.

Ergo hæ expressiones cognitæ

$$\left.\begin{array}{c} d \\ 1 + a + b + c + d \\ 16 + 8a + 4b + 2c + d \\ \&c. \end{array}\right\}$$ habebunt divisores in serie arithmetica progredientes.

Hoc ratiocinium secutus *Clairaut* acutissimus geometra tertiam *Algebræ* suæ partem copiose & solidissime elaboravit

§. 153.

sit radius circuli r, erit

Latus trigoni regularis circulo inscripti $= \sqrt{(3r^2)}$
— tetragoni —— —— $= \sqrt{(2r^2)}$
— pentagoni —— —— $= \sqrt{\frac{5}{2}r^2 - \frac{1}{2}\sqrt{5r^2}})$
— hexagoni —— —— $= r$
— octogoni —— —— $= \sqrt{(2r^2 - r\sqrt{2r^2})}$
— decagoni —— —— $= -\frac{1}{2}r + \frac{1}{2}\sqrt{5r^2}$

Sit

Sit latus polygonorum regularium l

erit radius circuli cui trigonum regulare inſcribitur $= \sqrt{\tfrac{1}{3}l^2}$

— — tetragonum — $= \sqrt{\tfrac{1}{2}l^2}$

— — pentagonum — $= \sqrt{\left(\dfrac{2l^2}{5-\sqrt{5}}\right)}$

— — hexagonum — $= l$

— — octogonum — $= l : \sqrt{(2-\sqrt{2})}$

— — decagonum — $= 2l : \sqrt{5} - 1$

Sit radius ſphæræ $= r$

Latus tetraedri ſphæræ inſcripti $= 2\sqrt{\tfrac{2}{3}r^2}$

— hexaedri — — $= 2\sqrt{(\tfrac{1}{3}r^2)}$

— octaedri — — $= 2\sqrt{(\tfrac{1}{2}r^2)}$

— dodecaedri — — $= \dfrac{-r + r\sqrt{5}}{\sqrt{3}}$

— icoſaedri — — $= \dfrac{2r(\sqrt{5}-1)}{\sqrt{(10-2\sqrt{5})}}$

Sit latus corporum regularium l

erit radius ſphæræ, cui tetraedrum inſcribitur $= \tfrac{1}{2}\sqrt{\tfrac{3}{2}l^2}$

— — cubus — $= \tfrac{1}{2}\sqrt{3l^2}$

— — octaedrum — $= l : \sqrt{2}$

— — dodecaedrum — $= l\sqrt{3} : -1 + \sqrt{5}$

— — icoſaedrum — $= l(\sqrt{10-2\sqrt{5}}) : \tfrac{1}{2}(\sqrt{5}-1)$

Element. Arithmet. Cap. IV. Probl. VIII. addantur exempla ſequentia in exercitium tyronum.

Quadrat.	Rad.
3894643909611477961	1973485219
77795775352911957243683441	2789189013479
144295118900085222589611477961	379861973485219

§. 463.

Æquatio generalis linearum ſecundi ordinis eſt

$$o = \Delta + \alpha x + \beta y + (\gamma x + \delta y)(\varepsilon x + \zeta y);$$

Δ denotat conſtantem

Ad portionis curvæ naturam inveſtigandam, quæ abſciſſæ x = infinitæ reſpondet, conſideretur membrum ultimum

$$(\gamma x + \delta y)(\varepsilon x + \zeta y)$$

Hujus vel uterque factor eſt imaginarius, vel uterque realis

I. Si uterque eſt imaginarius, curva nullum habebit ramum, qui in infinitum excurrit.

II. Si uterque factor fuerit realis, duo caſus ſunt evolvendi

a) ſi duo factores ſint inæquales

hoc caſu, facto $x = \infty$, alter factor finitum habere debet valorem, quia alias tota expreſſio non poſſet eſſe nihilo æqualis. Sit ergo $\gamma x + \delta y = A$ erit ob $x = \infty$, $\frac{y}{x} = -\frac{\gamma}{\delta}$, & $o = \Delta + (\alpha x + \beta y) + A(\varepsilon x + \zeta y)$ ubi ob Δ finitum fit $A = \frac{-\alpha x - \beta y}{\varepsilon x + \zeta y} = -\frac{\alpha\delta + \beta\gamma}{\varepsilon\delta - \zeta\gamma}$, natura hujus rami in infinitum excurrentis exprimitur hac æquatione $\gamma x + \delta y = \frac{\beta\gamma - \alpha\delta}{\delta\varepsilon - \gamma\zeta}$ quæ eſt pro linea recta, quæ ergo in infinitum producta cum curva confunditur, ideoque ejus eſt aſymptotos. Similiter alter factor $\varepsilon x + \zeta y$ monſtrabit aſymptoton, cujus erit æquatio $\varepsilon x + \zeta y = \frac{\beta\varepsilon - \alpha\zeta}{\gamma\zeta - \delta\varepsilon}$.

b) Si ambo factores ſint inter ſe æquales, ſeu $\varepsilon = \gamma$; & $\zeta = \delta$ fiet $o = \Delta + (\alpha x + \beta y) + (\gamma x + \delta y)^2$ & ob evaneſcens Δ præ in-

infinito, fiet $(\gamma x + \delta y)^2 + (\alpha x + \beta y) = 0$, quæ eſt æquatio ad parabolam, & oſtendit, curvam in infinitum eſſe parabolam. Erit ergo tota curva parabola.

Æquatio generalis pro lineis tertii ordinis,

$$0 = \Delta + (\alpha x + \beta y) + (\gamma x + \delta y)(\varepsilon x + \zeta y) + (\eta x + \theta x)(\iota x + \varkappa y)(\lambda x + \mu y)$$

quomodo curvæ portiones in infinitum abeuntes ſint comparatæ, ita definietur. Sumatur membrum ultimum in ſuos factores ſimplices reſolutum $(\eta x + \theta y)(\iota x + \varkappa y)(\lambda x + \mu y)$

I Vel erunt duo factores imaginarii, uniusque $\eta x + \theta y$ realis, debebit facto x vel $y = \infty$ hic factor eſſe finitus, ut à præcedente membro infinito hoc membrum tolli poſſit. Sit ergo $\eta x + \theta y = A$, erit $(\gamma x + \delta y)(\varepsilon x + \zeta y) + A(\iota x + \varkappa y)(\lambda x + \mu y) = 0$, hincque ob $\frac{x}{y} = -\frac{\theta}{\eta}$ erit $A = \frac{-(\delta\eta - \gamma\theta)(\zeta\eta - \varepsilon\theta)}{(\varkappa\eta - \iota\theta)(\mu\eta - \lambda\theta)}$ unde habetur æquatio pro aſymptoto una.

II. Sint omnes tres factores membri ultimi reales iique inter ſe

a) inæquales omnes, unusquisque præcedente modo tractatus dabit unam aſymptoton, unde curva habebit tres aſymptotos & ſex ramos in infinitum excurrentes,

b) ſint duo factores æquales, nempe $\iota = \eta$ & $\varkappa = \theta$, tertius factor $\lambda x + \mu y$ unam præbebit aſymptoton. Factores autem æquales poſito $x = \infty$ ponantur $(\eta x + \theta y)^2 = P$, erit $P(\lambda x + \mu y) + (\gamma x + \delta y)(\varepsilon x + \zeta y) + (\alpha x + \beta y) + \Delta = 0$.

α) Si membrum $(\gamma x + \delta y)(\varepsilon x + \zeta y)$ penitus deſit, fiet $P(\lambda x + \mu y) + \alpha x + \beta y = 0$ & $P = 0$, & ob $\frac{x}{y} = -\frac{\theta}{\eta}$ erit $P = -\frac{\alpha\theta - \beta\eta}{\lambda\theta + \mu\eta}$ ac propter duplicem valorem $\sqrt{P}$ curva tres habebit aſymptotas & ſex ramos in infinitum excurrentes.

β) Si membrum ſecundum non deſit, vel alter factor $\gamma x + \delta y$ eſt æqualis ipſi $\eta x + \theta y = \sqrt{P}$, vel neuter ipſi eſt æqualis 1) ſit $\eta x + \theta y = \gamma x + \delta y = \sqrt{P}$, erit $P(\lambda x + \mu y) + (\varepsilon x + \zeta y)$ $\sqrt{P}$

$\sqrt{P} + (\alpha x + \beta y) = 0$, & ob $\frac{y}{x} = -\frac{\eta}{\theta}$ erit $P(\lambda\theta - \mu\eta)$ $+ (\varepsilon\theta - \zeta\eta)\sqrt{P} + \alpha\theta - \beta\eta = 0$ unde duplex pro $\sqrt{P}$ nascitur valor, ex quo curva tres habebit asymptotas. 2) Si uterque factor membri secundi fuerit æqualis ipsi $\eta x + \theta y$, tum fiet $P(\lambda x + \mu y) + P + \alpha x + \beta y = 0$, hic P unicum habet valorem, unde curva habebit duas asymptotas. 3) Si neuter factor membri secundi fuerit æqualis $\eta x \pm \theta y$, tunc erit $P(\lambda x \pm \mu y) \pm (\gamma x \pm \delta y)(\varepsilon x \pm \zeta y) = 0$ & ob $\frac{y}{x} = -\frac{\eta}{\theta}$ erit $P = \frac{-(\gamma\theta - \delta y)(\varepsilon\theta - \zeta\eta)}{\lambda\theta - \mu\eta}\frac{x}{\theta}$ ideoque curva hoc casu præter unam asymptoton habebit duos ramos parabolicos in infinitum excurrentes

c) sint omnes tres factores membri supremi æquales & ponatur $(\eta x \pm \theta y)^3 = P^3$ erit $P^3 \pm (\gamma x \pm \delta y)(\varepsilon x \pm \zeta y) \pm (\alpha x \pm \beta y) = 0$.

α) desit membrum secundum, erit $P^3 \pm \alpha x \pm \beta y = 0$ ideoque $P^3 = -\alpha x - \beta y = (-\alpha\theta - \beta\eta)\frac{x}{\theta} = (\eta x \pm \theta y)^3$ & habebit curva duos ramos parabolicos secundi gradus in infinitum abeuntes.

β) Non desit membrum secundum, sit autem 1) ejus uterque factor $\eta x \pm \theta y = P$, erit iterum $P^3 \pm P^2 \pm \alpha x \pm \beta y = 0$, ideoque P^3 infinitum primi gradus & propterea $P^3 \pm \alpha x \pm \beta y = 0$, unde curva erit eadem quæ lit. α. 2) Sit unius tantum factor $\varepsilon x \pm \zeta y = \eta x \pm \theta y = P$, erit $P^3 \pm P(\gamma x \pm \delta y) \pm \alpha x \pm \beta y = 0$, erit ergo vel P finitum adeoque $P = -\frac{(\alpha x \pm \beta y)}{\gamma x \pm \delta y}$ vel $P^2 =$ infinito unius dimensionis, ut sit $P = \sqrt{-(\gamma\theta - \delta\eta)\frac{x}{\theta}} = \eta x \pm \theta y$ unde

de curva duos habebit ramos parabolicos, illo casu autem unam asymptoton. 3) Sit neuter factor membri secundi ipsi $\eta x \pm \theta y$ æqualis, erit $P^3 \pm (\gamma x \pm \delta y)(\varepsilon x \pm \zeta y) = 0$ & $P^3 = -(\gamma\theta - \delta\eta)$ $(\varepsilon\theta - \zeta y)\ \frac{xx}{\theta\theta} = (\eta x \pm \theta y)^3$ ex quo curva nullam habebit asymptoton at duos ramos parabolicos in infinitum excurrentes speciei $y^3 = ax^2$

Enumeratio generum curvarum trium dimensionum

I. Curvæ duobus ramis asymptoticis in infinitum excurrentes.

II. Curvæ duobus ramis parabolicis in infinitum abeuntes speciei $yy = ax$

III. Curvæ duobus ramis parabolicis in infinitum excurrentes, speciei $y^3 = a^2x$

IV. Curvæ duobus ramis parabolicis in infinitum abeuntes speciei $y^3 = ax^2$

V. Curvæ quatuor ramis asymptoticis in infinitum excurrentes.

VI. Curvæ quatuor ramis duobus asymptoticis & duobus parabolicis (speciei $yy = ax$) in infinitum excurrentes.

VII. Curvæ sex ramis asymptoticis in infinitum abeuntes.

§. 665.

Duplex est hujus æquationis & aliarum similium finis: $y^3 \pm a^2y - 2$ $\pm axy - x^3 = 0$. Vnus est, ut y per seriem eo magis convergentem determinetur, quo minor quantitas x assumitur; alter, ut y exprimat eo exactius, & eo convenientius veritati, quo major x assumitur. primo casu quæstio est, invenire y, si x = infinite parvo, in secundo si x = infinite magno. In utroque casu plures termini æquationis p reliquis evanescunt, & ii saltem, qui residui sunt, considerantur. Ita æquatione proposita, si x est infinite parvum, terminus x^3 evanes

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præ $2a^3$, & axy præ aay ita ut ſuperſit ſequens æquatio $y^3 + aay - 2a^3 = 0$ unde invenitur $y = a$ reliquis radicibus futuris imaginariis. Ergo ſi x eſt quantitas admodum parva, concluditur y = ſeriei convergenti, cujus primum membrum a & reliqui termini ſint dignitates τȣ x, quarum exponentes creſcant, adeoque ſumetur $y = a + Ax + Bx^2 + Cx^3$ &c. quæ ſeries ſubſtituta in æquatione data exhibet

$$\left.\begin{array}{llll} y^3 = a^3 + 3aaAx & + 3aaBx^2 & + 3aaC & \\ & + 3aAA & + 6aAB & x^3 \\ & & + A^3 & \\ a^2y = a^3 + aaAx & + aaBx^2 & + aaCx^3 & \\ axy = \quad\;\; aax & + aAx^2 & + aBx^3 & \\ -2a^3 = -2a^3 & & & \\ -x^3 = & & -x^3 & \end{array}\right\} \text{\&c.} = 0$$

$$4aaA + aa = 0, \text{ unde } A = -\tfrac{1}{4}$$

$$\frac{3a}{16} + 4a^2B - \tfrac{1}{4}a = 0, \text{ hinc } B = \frac{1}{64a}$$

$$4a^2C - \frac{131}{128} = 0, \text{ quare } C = \frac{131}{512a^2}$$

Erit ergo $y = a - \frac{x}{4} + \frac{x^2}{64a} + \frac{131x^3}{512a^2}$

quæ ſeries eo veritati propiorem valorem τȣ y præbet quo minor eſt quantitas x.

In ſecundo caſu ubi y quæritur, ſi x ſit quantitas notabilis, ponatur $x = \infty$, & in æquatione propoſita terminus $2a^3$ evaneſcet præ x^3 & aay præ axy hinc æquatio erit $y^3 + axy - x^3 = 0$, jam judicandum eſt an y ſit infinitum ejusdem vel ſuperioris vel inferioris ordinis quam x, hic ſtatim apparet, quod y non ſit ordinis ſuperioris, namque eſſe deberet $y^3 + axy > x^3$ neque etiam ordinis inferioris, quia alias foret $y^3 + axy$

$< x^3$ adeoque y eſt ejusdem ordinis cujus eſt x, quare terminus axy evaneſcit præ duobus reliquis, ita ut ſit $y^3 - x^3 = 0$, hinc $y = x$ valor verus τȣ y ſi $x = \infty$ ſi vero x non ſumatur pro quantitate infinite magna, ſed ſaltem admodum magna, adjiciendi adhuc ſunt quidam termini vel poteſtates quantitatis x quarum exponentes decreſcunt, ita ut quilibet conſequens evaneſcat præ antecedente. Erit itaque forma æquationis

$y = x + A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{x^3} +$ &c. hinc æquatio propoſita abibit

$$\left.\begin{array}{rlllll}
y^3 = & x^3 & + 3Axx & + 3Bx & + 3C & \\
 & & & + 3A^2 & + 6AB & \\
 & & & & + A^3 & \\
 & & & & & \text{\&c.} \\
a^2y = & & & + a^2x & + a^2A & \\
axy = & & ax^2 & + aA & + aB & \\
-x^3 = & -1 & & & & \\
-2a^3 = & & & & -2a^3 &
\end{array}\right\} = 0$$

$$3A + a = 0, \text{ hinc } A = -\frac{a}{3}$$

$$3B + \frac{a^2}{3} + a^2 - \frac{a^2}{3} = 0 \ \& \ B = -\frac{a^2}{3}$$

$$3C + \frac{2a^3}{3} - \frac{a^3}{27} - 2a^3 - \frac{a^3}{3} - \frac{a^3}{3} = 0 \ \& \ C = \frac{55}{81}a^3$$

Cum hæc procedendi & judicandi methodus in æquationibus maxime compoſitis perquam difficilis fieri poſſit, Newtonus ope ſui parallelogrammi eam faciliorem reddere anniſus eſt.

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x^3	x^3y	x^3y^2	x^3y^3
x^2	x^2y	x^2y^2	x^2y^3
x	xy	xy^2	xy^3
1	y	y^2	y^3

Jam notandum eſt, ſi x eſt quantitas infinite parva, in qualibet columna termini ſuperiores evaneſcunt præ inferioribus, & vice verſa ſi x eſt quantitas infinite magna, termini inferiores evaneſcunt præ ſuperioribus. Terminis itaque æquationis alicujus propoſitæ ita diſpoſitis, in utroque caſu ſtatim apparebit quinam termini negligi poſſint, ut primus ſeriei formandæ terminus inveniatur: in primo caſu inferiores termini ſerierum verticalium, in ſecundo ſuperiores adhibentur, qui etiam homogenei conſiderantur. Conſiderandum adhuc eſt ſi v. g. x^3 & xy^3 ſint termini homogenei, & ducatur linea recta à x^3 ad xy^3 omnes termini ex una parte hujus lineæ erunt infinite majores, ex altera infinite minores, ſed ſi linea recta per centrum alicujus quadrati adhuc tranſiret, terminus ejus etiam foret homogeneus. v. g. ſi recta à x^3y ad xy^3 duceretur tranſiret ea per x^2y^2 qui adeoque foret homogeneus τοῖς x^3y & xy^3, adeoque nec infinite major nec infinite minor illis.

	x^6y					
x^5			x^5y^3			
				x^4y^4	x^4y^5	
x^3		x^3y^2				
x^2						x^2y^6
	xy			xy^4		
1		y^2				

Sit e. g. propoſita æquatio $a+bx^2+cxy+dy^2+ex^3+fx^5+gx^3y^2+hxy^4+ix^6y+kx^5y^3+lx^4y^4+mx^5y^6+nx^4y_5$ $=0$; diſpoſitis terminis in parallelogrammo uti vides. Si igitur quæratur valor τȢ y, ſi $x=0$ infin. parv. non niſi termini inferiores conſiderantur, hinc duæ ſunt ſolutiones, prima ſumitur

$a+dy^2$

$a + dy^2 = 0$, & valor $\tau\breve{\varepsilon}$ $y = \sqrt{-\frac{a}{d}}$; secunda solutio $dy^2 + bxy^4 + mx^2y^6 = 0$, hinc vel $y = 0$, vel $d + bxy^2 + mx^2y^4 = 0$ i. e. $y = \frac{\sqrt{(-b + \sqrt{(b^2 - 4dm)})\, x^{-\frac{1}{2}}}}{\sqrt{2m}}$. Tres igitur casus sunt evolvendi

1) $y = A + Bx + Cx^2 +$ &c.

2) $y = Ax + Bx^2 + Cx^3 +$ &c.

3) $y = Ax^{-\frac{1}{2}} + Bx^{\frac{1}{2}} + Cx^{\frac{3}{2}}$ &c.

quod in quolibet casu facile animadvertitur. Sed si quæratur valor $\tau\breve{\varepsilon}$ y, si x est admodum magna quantitas, ponatur $x = \infty$, recta à terminis superioribus ad inferiores ducitur, unde tres nascuntur æquationes accidere potest, ut hæ series crescant x^2, x^3,

1) $fx^5 + ix^6y = 0$, unde $y = \frac{-f}{ix}$, & series ipsa $y = \frac{-f}{ix} + \frac{B}{x^2} + Cx^{-3} + Dx^{-4}$ &c.

2) $ix^6y + kx^5y^3 + nx^4y^5 = 0$, hinc vel $y = 0$, vel $ix^2y + kxy^3 + ny^4 = 0$, & $y = Ax^{\frac{1}{2}}$, & ipsa series $y = Ax^{\frac{1}{2}} + Bx^{-\frac{1}{2}} + Cx^{-\frac{3}{2}}$

3) $nx^4y^5 + mx^2y^6 = 0$ vel $nx^2 + my = 0$, hinc $y = -\frac{nx^2}{m}$ & ipsa series $y = -\frac{nx^2}{m} + Ax + B + \frac{C}{x} + \frac{D}{x^2}$ &c.

Hæ resolutiones quam maxime locum habent, si forma lineæ curvæ investigatur, ubi abscissa est vel infinite magna vel infinite parva.

THESES.

THESES.

I.

In magnete dantur interſtitia eo modo diſpoſita ut ductus forment, qui ſubtiliori materiæ tranſitum ſecundum determinatam directionem concedunt, ſecundum aliam recuſant.

II.

Eodem modo poteſt ferrum diſponi, ita ud id loco magnetis naturalis poſſit adhiberi.

III.

Vapores ſunt bullulæ pellicula aquea conſtantes & interne aere repletæ.

IV.

Hinc elevatio vaporum bene ex notiſſimo principio hydroſtatico explicari poteſt: *Levius fluidum aſcendit in ſpecifice graviori ſeu denſori. gravius deſcendit in leviore ſeu rariore.*

V.

Ros male inter meteora refertur.

VI.

Cur quidam lacus in Italia tempeſtate pluvia evacuati aquant, ex doctrina de ſiphonibus, quorum effluxum preſſio aeris efficit, bene illuſtratur.

VII.

Abſurdiſſima eſt hypotheſis Abbatis de *Brancas*, qui ſtellas fixas à ſole noſtro illuminari contendit.

VIII.

Apparitiones ſtellarum variabilium optime explicantur, ſi eadem earum figura, quæ lentibus eſt, eſſe ſupponitur.

IX.

Ratio cur radii ſolares magis ad perpendiculum refringantur in medio denſiore quam rariore, eſt quia ab iſto magis attrahuntur quam ab hoc.

X.

Hinc celeritas lucis in diverſis mediis eſt in ratione inverſa denſitatis mediorum, id quod bene demonſtratur methodo, qua MAUPERTUIS utitur.

XI.

Abſurde aſſumunt Platonici & Galenus radios quibus objecta videmus, ex oculo videntis promanare.

XII.

Sit f locus imaginum in ſpeculo ſphærico cujus radius r & d diſtantia objecti ante ſpeculum, erit $f = \frac{dr}{2d-r}$ ſi $r = \infty$ habe-tur ſpeculum planum, quare hic $f = - d$.

XIII.

Non datur methodus cujus ope exacte dolia ad aliquam pa-tem evacuata menſurari poſſint.

XI

XIV.

Lunæ jure atmoſphæra videtur tribui.

XV.

Motus rotatorius corporum cœleſtium circa ſuos axes bene ex obſervatione macularum in earum ſuperficiebus poſitarum concluditur.

XVI.

Cometæ non differunt à planetis, niſi quod eorum orbitæ ſint aliæ ſectiones conicæ.

XVII.

Si terra 18es celerius circa axem volveretur, ad atomum redigeretur.

XVIII.

Si gravitas aſſumitur proportionata diſtantiæ à centro, & vis centrifuga æqualis gravitati, terra eſſet planum circulare, eſt enim hoc caſu

Diameter æquatoris ad axem revolutionis ut $\sqrt{p}$ ad $\sqrt{(p-f)}$ denotante p gravitatem, f vim centrifugam.

XIX.

Syſtema vulgare Briggianum reliquis poſſibilibus jure præfertur.

XX.

Perfectum vacuum effici non poteſt.

XXI.

Fulgur & tonitru bene ab electricitate explicantur.

XXII.

XXII.

Capite majus ſpatium ambulando abſolvimus quam pedibus.

XXIII.

x^0 eſt = 1, quicunque numerus pro x ponatur.

XXIV.

Cur tempeſtate nebuloſa Mercurius in Barometro nonnunquam aſcendat, probabiliſſime explicuit HALLEIUS cauſam hujus phænomeni à ventis horizontaliter flantibus deducens.

XXV.

Cauſſa fractionis radii ad perpendiculum eſt attractio medii denſioris.

XXVI.

Remi bene inter vectes referuntur. Vid. Phyſ. KRAFFT. P. II. §. 32.

XXVII.

Libra male inter machinas ſimplices refertur.

F I N I S.